Extended Standard-Model by the Coxeter element of the affine Weyl group $\tilde{E}8$

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Abstract

In this paper i will give an extension of the known Standard-Model.The shape of the extension is not arbitrary choosen.The shape explains gravity and more.
I show that the symmetries generated by the coxeter-element of the affine Weyl group $\tilde{E}8$ which is the affine extension of the well known exceptional group E8 is a candidate which explains open questions like dark matter and gravity.

Keywords: *dark matter; standardmodel;forces; gravity*

1. Introduction

The Standard Model of particle physics is the theory describing three of the four known fundamental forces which are electromagnetic, weak and strong interactions.
The fourth known force gravity is not included until today.
In this paper i want to filling the gap and furthermore i want to show candidates for dark matter particles. In the following text i will first explain what is a Coxeter group and what is a Coxeter element.Then i will show how the Extendet Standard Model is produced by a Coxeter element of the affine Weyl group $\tilde{E}8$.
Last but not least i will show a backgroundfield and a potential for the extension which is similar to the Higgsfield and the Higgspotential.

1.1. <u>The Extended Standard-Model short ESM</u>

$$SU(5)_s \times SU(3)_c \times SU(2)_L \times U(1)_y \times SE(1)_t$$

$s ...sense\ charges\ (sight, smell, touch, taste, hearing)$
$c ...color\ charges\ (red, green, blue)$
$L ...weak\ isospin$
$y ...weak\ hypercharge$
$t ...translation\ of\ speed\ and\ therefore\ acceleration$

$SU(n) ...special\ unitaery\ group$
$U(1) ...unitaery\ group$
$SE(1) ...special\ euclidian\ group$

1.1.1. Deduction of the ESM by a special Coxeter element

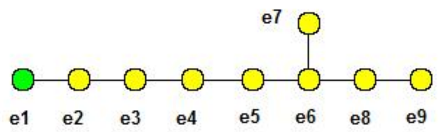
Def.1 Coxeter group

The group is named after H.S.M. Coxeter a british mathematician (1907-2003)
Formally, a Coxeter group W can be defined as a group with the presentation

$$W = \langle r_1, r_2, ..., r_n | (r_i r_j)^{m_{ij}} = 1 \rangle \quad r_i ... reflections \tag{1}$$

with $m_{ii} = 1$ *and* $m_{ij} = m_{ji} \geq 2$ *for* $i \neq j$
the condition $m_{ij} = \infty$ *means no relation of the form* $(r_i r_j)^m$ *should be imposed.*
The pair (W, S) *where* W *is a Coxeter group with generators* $S = \{r_1, r_2, ..., r_n\}$
is called a Coxeter system.

Example:The coxeterdiagram of the affine coxetergroup $\tilde{E}8$



$\tilde{\mathbf{E}}_8$

The reason why we use Coxeter groups which a more abstract as reflection groups is because we want to use the results of the Coxeter theory.
This is possible because Coxeter shows that every reflection group is a Coxeter group.
Every (affine) Weyl group is a (affine) Coxeter group.

Def.2 Coxeter element

A Coxeter element is a product of all simple reflections
For example $r_1.r_2....r_{n-1}.r_n$ or $r_2.r_{n-1}....r_1$
All permutations of the simple reflections are Coxeter elements.

Def.3 Coxeter polynomial

A Coxeter polynomial for the Coxeter system (W, S) is the characteristic polynomial of a Coxeter element.
Example 1: Coxeter polynomial of the affine Weyl group $\tilde{A}_2$

$$P(\lambda) = \frac{\lambda^2 - 1}{\lambda - 1}.(\lambda - 1)^2 = \Phi_2.\Phi_1^2 \tag{2}$$

Eigenvalues are $\lambda_1 = -1$ and $\lambda_2 = 1$

Example 2: Coxeter polynomial of the affine Weyl group $\tilde{E}_8$

$$P(\lambda) = \frac{\lambda^5 - 1}{\lambda - 1}.\frac{\lambda^3 - 1}{\lambda - 1}.\frac{\lambda^2 - 1}{\lambda - 1}.(\lambda - 1)^2 = \Phi_5.\Phi_3.\Phi_2.\Phi_1^2 \tag{3}$$

Φ_n are the so called cyclotomic factors because the zeropoints of such a factor have the shape $e^{\frac{k2\pi i}{n}}$ k=1,...,n-1

see References $[DE01]$ Site 15/Section 5/Table 2

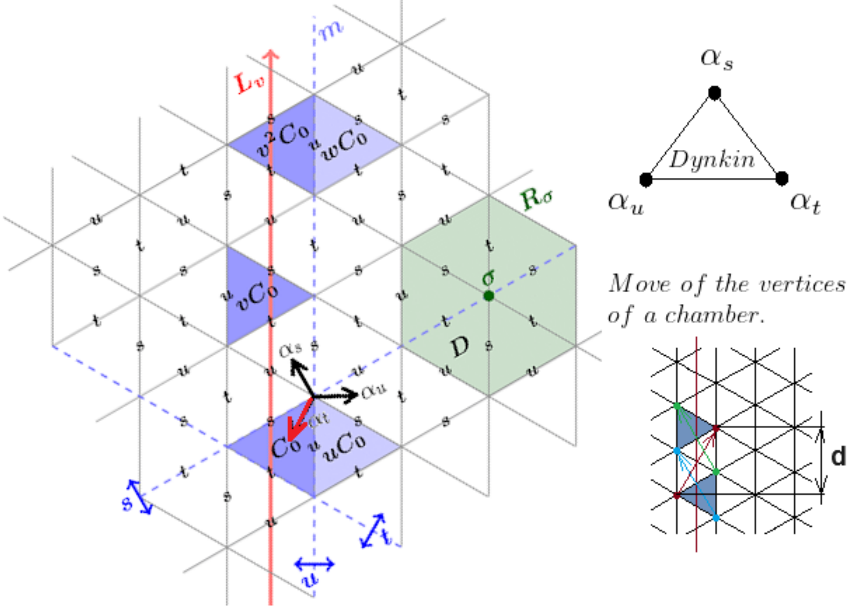
Def.4 Coxeter number h

the Coxeter number h is the order of a Coxeter element of an irreducible Coxeter group, hence also of a root system or its Weyl group.
For example the exceptional group E8 with Coxeter element $v = r_1.r_2.r_3.r_4.r_5.r_6.r_7.r_8$ with $r_1, ..., r_8$ the generators of E8.
The Coxeter number of E8 ist 30 that means $C_e^{30} = e = identity$.
There are different definitions for the Coxeter number.
Another definition of the Coxeter number is:
The dimension of the corresponding Lie algebra is n(h + 1), where n is the rank of the Coxeter group and h is the Coxeter number.
For example the reflection group A_2 where the corresponding Lie algebra is $\mathfrak{su}(3)$ and the Lie group is SU(3).
We know SU(3) has 8 generators which is the dimension of the Lie algebra $\mathfrak{su}(3)$ thus $8 = n(h+1) = 2(h+1)$ then $h = 3$.

Def.5 affine Coxeter groups from the finite Coxeter groups

Suppose R is an irreducible root system of rank r>1 and let $\alpha_1, \ldots, \alpha_r$ be a collection of simple roots. Let, also, α_{r+1} denote the highest root.
Then the affine Coxeter group is generated by the ordinary (linear) reflections about the hyperplanes perpendicular to $\alpha_1, \ldots, \alpha_r$, together with an affine reflection about a translate of the hyperplane perpendicular to α_{r+1}.

For example the affine Coxeter group $\tilde{A}_2$ generated by the Coxeter group A_2.



Simple roots of A_2 are α_u, α_s , the highest root α_t in red and the fundamental domain or chamber C_0 in violet.

One Coxeter element is $v = s.u.t$ and the action of this Coxeter element v moves the chamber C_0 to $v.C_0$.Doing it one more time the action is v^2 and it moves the chamber C_0 to $v^2.C_0$.
We can see in the picture that the action of the Coxeterelement moves the chamber along the red line L_v which is named Coxeter axis and reflect the chamber on the red line L_v.

This is a so called glidereflection.
The double action of the Coxeter element moves the chamber the double way along the red line L_v. Remark h=2 is the coxeternumber of $\tilde{A}_2$.

In this representation the Coxeter element acts as an affine euclidian map
$\alpha : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$
$\vec{x} \longmapsto A.\vec{x} + \vec{d}$

We set the origin anywhere on the red translationline then the map is

$$\vec{y} = A.\vec{x} + \vec{d} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ d \end{pmatrix} \; \textit{this is a reflection and a translation} \tag{4}$$

Then the augmented representation is

$$\begin{pmatrix} y_1 \\ y_2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & d \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix} \tag{5}$$

The characteristic polynomial as seen in (2) is

$$P(\lambda) = \frac{\lambda^2 - 1}{\lambda - 1}.(\lambda - 1)^2 = \Phi_2.\Phi_1^2 \tag{6}$$

Eigenvalues are $\lambda_1 = -1$ and $\lambda_2 = 1$
Remark: the algebraic multiplicity of λ_2 is 2 and the geometric multiplicity is 1!
We name the eigenvalue λ_1 which comes from the cyclotomic factor Φ_2 λ_h a horizontal eigenvalue because it belongs to a horizontal eigenvector a horizontal root.
This roots are orthogonal to the translation axis L_v.
In this example the eigenvector EV_h to the eigenvalue $\lambda_h = -1$ is

$$EV_h = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{7}$$

Furthermore we name the eigenvalue λ_2 for one time λ_v as vertical eigenvalue and for one time λ_t as translation eigenvalue.
In this example the eigenvector to $\lambda_v = \lambda_t = 1$ is

$$EV_v = EV_t = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{8}$$

More details for this see References [JM01]
What i want to show by this example is that the action of a Coxeter element (affine map in our example) can be understood by the terms the so called cyclotomic factors $\Phi_2.\Phi_1^2$ of the Coxeter polynomial.
One Φ_1 of the term Φ_1^2 is responsible for the translation along the axis L_v.
The rest $\Phi_2.\Phi_1$ is responsible for the linear part of the affine map.
The horizontal eigenvector EV_h with eigenvalue $\lambda_h = -1$ is the simple root of the group A_1 and therefore corresponds to $\mathfrak{su}(2)$ $(SU(2))$.
The vertical eigenvector EV_v with eigenvalue $\lambda_v = 1$ is a generator in $\mathbb{R}$ and therefore is corresponding to $\mathfrak{u}(1)$ $(U(1))$.
The translation eigenvector EV_t with eigenvalue $\lambda_t = 1$ is the generator of translations in $\mathbb{R}$ and

therefore is corresponding to the special euclidian group SE(1).

Then our total Lie algebra is $\mathfrak{su}(2) \oplus \mathfrak{u}(1) \oplus \mathbb{R}$ and therefore the total Lie group is

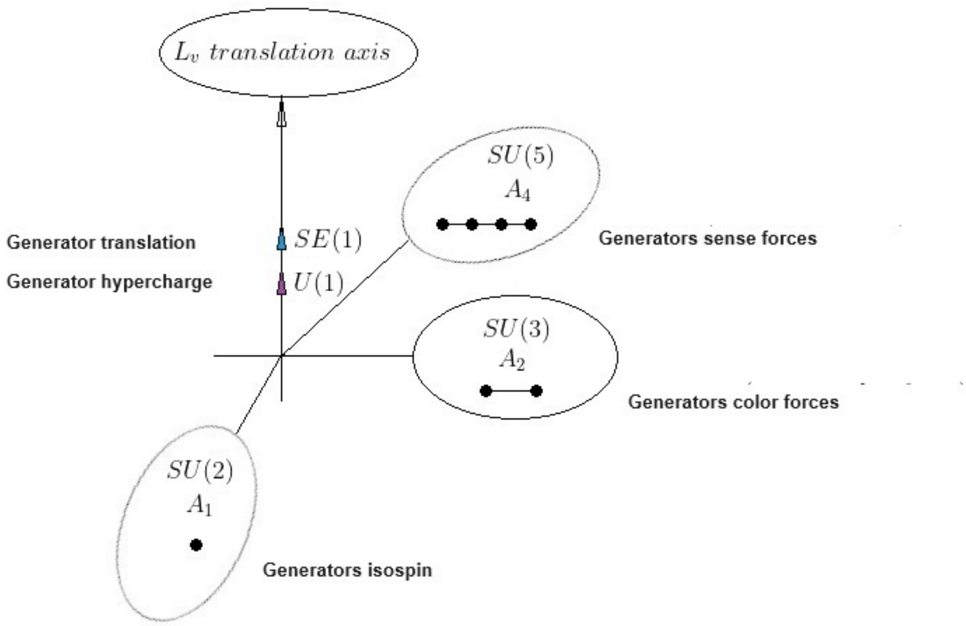
$SU(2) \times U(1) \times SE(1)$
| | |
$\Phi_2 \times \Phi_1 \times \Phi_1$ = Coxeter polynomial (characteristic polynomial) of $\tilde{A}_2$

1)It is well known that the cyclotomic factor Φ_p is the characteristic polynomial of the group A_{p-1} if p is prime.
2)And it also well known that for the Weyl group A_{p-1} the corresponding Lie algebra is $\mathfrak{su}(\mathfrak{p})$.
Then the eigenvalues and eigenvectors of a Coxeter element of $\tilde{E}8$ corresponds in the same manner as above to the Lie algebra $\mathfrak{su}(5) \oplus \mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1) \oplus \mathbb{R}$. This Lie algebra composition generates the Lie group below which is our extended Standard model.

$SU(5) \times SU(3) \times SU(2) \times U(1) \times SE(1)$
| | | | |
$\Phi_5 \times \Phi_3 \times \Phi_2 \times \Phi_1 \times \Phi_1$ = Coxeter polynomial of $\tilde{E}8$ see (3)

This expands the Standard model by the components $SU(5) \times SE(1)$.

Schematic representation:



For completeness we want to show the eigenvalues and eigenvectors of the action of the Coxeter element.Our information we got from Reference [JM01] example 11.8
The Coxeter element can be represented by an affine map.Some eigenvalues are complex therefore

$\alpha : \mathbb{C}^8 \longrightarrow \mathbb{C}^8$
$\vec{x} \longmapsto A.\vec{x} + \vec{d}$

The translation axis L_v is

$$EV_t = L_v = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 3 \\ -3 \\ 2 \\ 2 \end{pmatrix} \quad eigenvalue\ \lambda_t = 1 \tag{9}$$

and the eigenvectors the simple roots of A_1, A_2, A_4 are

$$set\ of\ eigenvectors\ EV\ of\ A_1 = EV_{A_1} = \{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}\}\ eigenvalue\ \lambda_v = e^{\frac{2\pi i}{2}} = -1 \tag{10}$$

and

$$EV_{A_2} = \{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}\}\ eigenvalues\ \lambda_{A_2}(k) = e^{\frac{k2\pi i}{3}} \quad k = 1, 2 \tag{11}$$

and

$$EV_{A_4} = \{\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \end{pmatrix}\}\ eigenvalues\ \lambda_{A_4}(k) = e^{\frac{k2\pi i}{5}} \quad k = 1, 2, 3, 4 \tag{12}$$

2. Lagrange Density by the additional components $SU(5) \times SE(1)$

$$\mathcal{L}_{OQF} = (D^{\mu}\phi)^{\dagger}(D_{\mu}\phi) - V(\phi) \qquad (energydensity)^2$$

OQF...OctoQuintenField

$D_{\mu}\phi = (\partial_{\mu} + ig\tau_{ij}W^{ij}_{\mu} + ig^{'}I_5B^0_{\mu})\phi$ *covariant derivation with coupling constants* $g, g^{'}$

$V(\phi)$*...Evolution Potential on the OctoQuintenField*

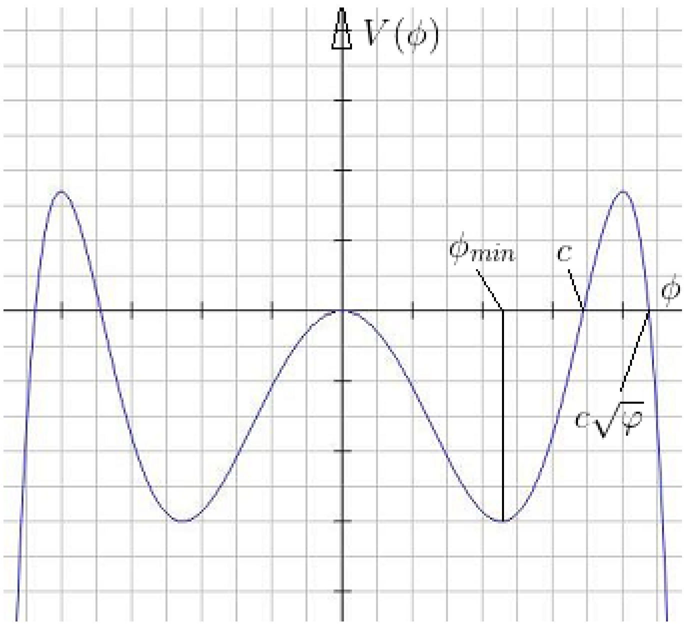
Similar to the Higgs mechanism where a backgroundfield the so called Higgs field with the Higgs potential on it brakes the symmetry group $SU(2)_L \times U(1)_y$ down to $U(1)_{em}$ and giving mass to the $SU(2)$ bosons we want to give the $SU(5)$ bosons mass by a backgroundfield which we call OctoQuintenfield (OQF).The potential on it has the following shape and is called the Evolutionpotential.More details see References [RK01].

The equation is an $energydensity^2$ potential with speed as variable.

$$V(\phi) = (\frac{\Lambda.c^4}{8\pi G})^2.\left(-\frac{\varphi^3}{\varphi^3+1}(\frac{\mid\phi\mid}{c})^2 + (\frac{\mid\phi\mid}{c})^4 - \frac{1}{\varphi^3+1}(\frac{\mid\phi\mid}{c})^8\right) \tag{13}$$

$speed\ \phi \in \mathbb{O}^5 \times i\mathbb{R}^8 = Octonions^5 \times i\mathbb{R}^8$
and $\mid\phi\mid \leqq c\sqrt{\varphi}$
$\mid\phi\mid = \sqrt{\phi^{\dagger}\phi}$
c*...speed of light*
Λ*... cosmological constant*
G*...gravitation constant*
φ*...golden mean* $1,618033...$

2.1. Picture of the equation



the potential is zero on the spheres or shells

$\mid \phi \mid = 0, \mid \phi \mid = c \; and \; \mid \phi \mid = c\sqrt{\varphi}$

1) The $4 + 20 = 24$ generators of the Lie algebra $\mathfrak{su}(5)$ are

4 abelian generators and 20 non abelian generators
24 linearly independent 5×5 traceless Hermitian matrices
The four blue matrices are the abelian generators.

$$\begin{pmatrix}1&0&0&0&0\\0&-1&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix}\begin{pmatrix}0&1&0&0&0\\1&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix}\begin{pmatrix}0&0&1&0&0\\0&0&0&0&0\\1&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix}\begin{pmatrix}0&0&0&1&0\\0&0&0&0&0\\0&0&0&0&0\\1&0&0&0&0\\0&0&0&0&0\end{pmatrix}\begin{pmatrix}0&0&0&0&1\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\1&0&0&0&0\end{pmatrix}$$

$$\begin{pmatrix}0&-i&0&0&0\\i&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix}\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&-2&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix}\begin{pmatrix}0&0&0&0&0\\0&0&1&0&0\\0&1&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix}\begin{pmatrix}0&0&0&0&0\\0&0&0&1&0\\0&0&0&0&0\\0&1&0&0&0\\0&0&0&0&0\end{pmatrix}\begin{pmatrix}0&0&0&0&0\\0&0&0&0&1\\0&0&0&0&0\\0&0&0&0&0\\0&1&0&0&0\end{pmatrix}$$

$$\begin{pmatrix}0&0&-i&0&0\\0&0&0&0&0\\i&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix}\begin{pmatrix}0&0&0&0&0\\0&0&-i&0&0\\0&i&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix}\frac{1}{\sqrt{6}}\begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&-3&0\\0&0&0&0&0\end{pmatrix}\begin{pmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&1&0\\0&0&1&0&0\\0&0&0&0&0\end{pmatrix}\begin{pmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&1\\0&0&0&0&0\\0&0&1&0&0\end{pmatrix}$$

$$\begin{pmatrix}0&0&0&-i&0\\0&0&0&0&0\\0&0&0&0&0\\i&0&0&0&0\\0&0&0&0&0\end{pmatrix}\begin{pmatrix}0&0&0&0&0\\0&0&0&-i&0\\0&0&0&0&0\\0&i&0&0&0\\0&0&0&0&0\end{pmatrix}\begin{pmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&-i&0\\0&0&i&0&0\\0&0&0&0&0\end{pmatrix}\frac{1}{\sqrt{10}}\begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&-4\end{pmatrix}\begin{pmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&1\\0&0&0&1&0\end{pmatrix}$$

$$\begin{pmatrix}0&0&0&0&-i\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\i&0&0&0&0\end{pmatrix}\begin{pmatrix}0&0&0&0&0\\0&0&0&0&-i\\0&0&0&0&0\\0&0&0&0&0\\0&i&0&0&0\end{pmatrix}\begin{pmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&-i\\0&0&0&0&0\\0&0&i&0&0\end{pmatrix}\begin{pmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&-i\\0&0&0&i&0\end{pmatrix}$$

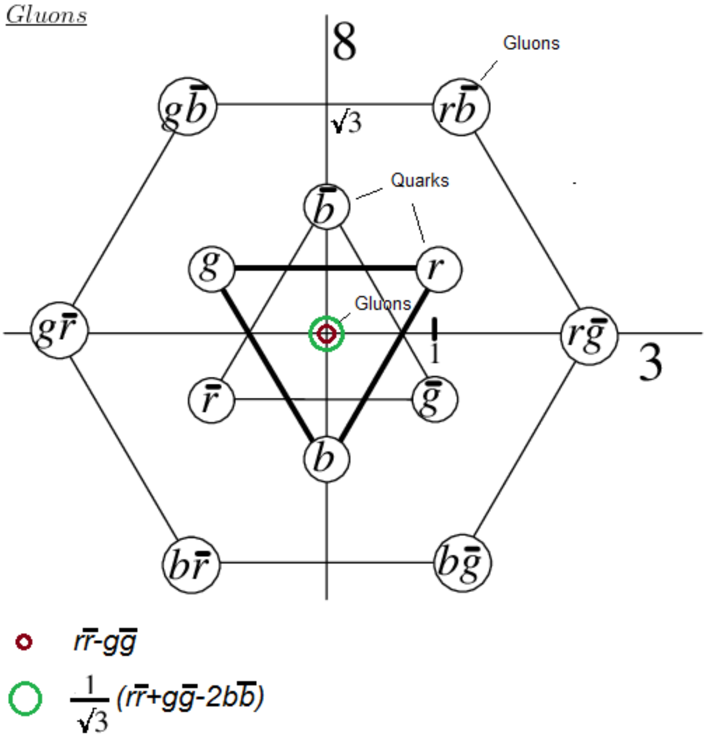
For easier handling we enumerate the generators by the following map

$$\begin{pmatrix}\tau_{11}&\tau_{21}&\tau_{31}&\tau_{41}&\tau_{51}\\\tau_{12}&\tau_{22}&\tau_{32}&\tau_{42}&\tau_{52}\\\tau_{13}&\tau_{23}&\tau_{33}&\tau_{43}&\tau_{53}\\\tau_{14}&\tau_{24}&\tau_{34}&\tau_{44}&\tau_{54}\\\tau_{15}&\tau_{25}&\tau_{35}&\tau_{45}&\times\end{pmatrix} \tag{14}$$

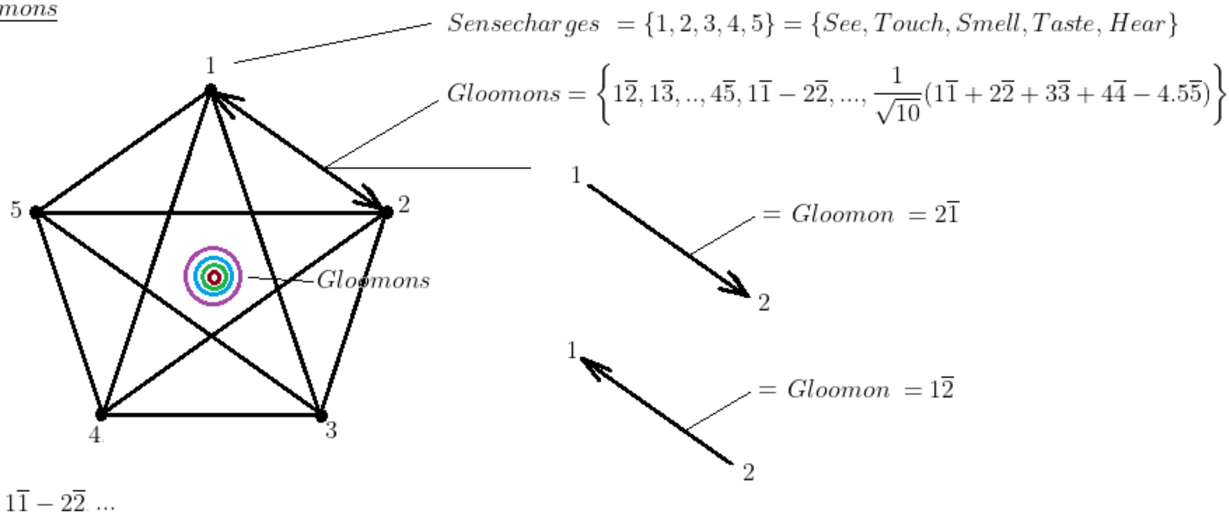
Similar to the groups SU(2) and SU(3) the generators corresponding to vector bosons.
$SU(2)$ $2^2 - 1 = 3$ vector bosons are W^+, W^- and Z^0
$SU(3)$ $3^2 - 1 = 8$ vector bosons are Gluons
The $SU(5)$ $5^2 - 1 = 24$ vector bosons we name Gloomons and the five charges we call senses.

A picture to see the similarities between $SU(3)$ and $SU(5)$.
Hint:the Gluons can be represented in $\mathbb{R}^2$ and the Gloomons in $\mathbb{R}^4$.Therefore it is not possible to draw the same picture.

Gluons



Gloomons



Now we say that the Gloomons get a mass similar like the W^+, W^- *and* Z^0 bosons by coupling to a backgroundfield.
Instead of 2 charges we have 5 therefore it is instead of a doublet a quintet.
And instead of a complex row we use a octonionic row.Then we get a backroundfield which we call OctoQuintenfield (short OQF).

representation of the definitionrange as (extented) octonionic vector $\phi =$

$$\begin{pmatrix} (\phi_{2\bar{1}} + i\phi_0).1 & +\phi_{01}i_1 & +\phi_{02}i_2 & +\phi_{03}i_3 & +\phi_{\overline{21}}i_4 & +\phi_{\overline{23}}i_5 & +\phi_{\overline{24}}i_6 & +\phi_{\overline{25}}i_7 \\ \phi_{10}.1 & +(\phi_{3\bar{1}} + i\phi_1).i_1 & +\phi_{12}i_2 & +\phi_{13}i_3 & +\phi_{\overline{32}}i_4 & +\phi_{\overline{31}}i_5 & +\phi_{\overline{34}}i_6 & +\phi_{\overline{35}}i_7 \\ \phi_{20}.1 & +\phi_{21}i_1 & +(\phi_{4\bar{1}} + i\phi_2).i_2 & +\phi_{23}i_3 & +\phi_{\overline{42}}i_4 & +\phi_{\overline{43}}i_5 & +\phi_{\overline{41}}i_6 & +\phi_{\overline{45}}i_7 \\ \phi_{30}.1 & +\phi_{31}i_1 & +\phi_{32}i_2 & +(\phi_{5\bar{1}} + i\phi_3).i_3 & +\phi_{\overline{52}}i_4 & +\phi_{\overline{53}}i_5 & +\phi_{\overline{54}}i_6 & +\phi_{\overline{51}}i_7 \\ (\phi_{00} + i\phi_{\overline{00}}).1 & +(\phi_{11} + i\phi_{\overline{11}}).i_1 & +(\phi_{22} + i\phi_{\overline{22}}).i_2 & +(\phi_{33} + i\phi_{\overline{33}}).i_3 & +\phi_{44}i_4 & +\phi_{55}i_5 & +\phi_{66}i_6 & +\phi_{77}i_7 \end{pmatrix} \quad (15)$$

with $\phi_{xx} \in \mathbb{R}$
Dimension of the definitionrange = 5 x 8 + 8 = 48
The index xx of the ϕ_{xx} gives a map to the bosons,spacetime curvature and more.
The 19 blue DOF and one violet DOF we associate to spacetime curvature by the following way.
The DOF's are speed.Curvature and speed is connected by the formular which is similar to the Bernoulli equation $P = Const.v^2$

$$\phi^2.r^2 = \frac{G\hbar}{c} \tag{16}$$

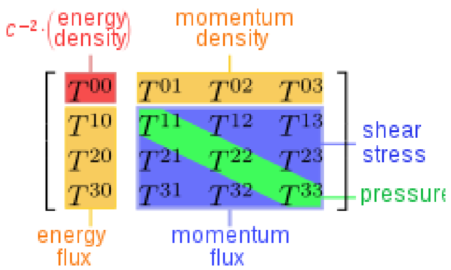
$r ... radius\ of\ curvature$

The equation (16) shows that if we have curvature we also have speed.Speed of what? It's the speed of spacetime itself.

Symmetric curvature tensor by the blue fields:

$$K = \frac{c}{G\hbar}.\begin{pmatrix} (\phi_{00}+i\phi_{\overline{00}})^2.1.1 & \phi_{01}\phi_{10}.1.i_1 & \phi_{02}\phi_{20}.1.i_2 & \phi_{03}\phi_{30}.1.i_3 \\ \phi_{01}\phi_{10}.i_1.1 & (\phi_{11}+i\phi_{\overline{11}})^2.i_1.i_1 & \phi_{12}\phi_{21}.i_1.i_2 & \phi_{13}\phi_{31}.i_1.i_3 \\ \phi_{02}\phi_{20}.i_2.1 & \phi_{12}\phi_{21}.i_2.i_1 & (\phi_{22}+i\phi_{\overline{22}})^2.i_2.i_2 & \phi_{23}\phi_{32}.i_2.i_3 \\ \phi_{03}\phi_{30}.i_3.1 & \phi_{13}\phi_{31}.i_3.i_1 & \phi_{23}\phi_{32}.i_3.i_2 & (\phi_{33}+i\phi_{\overline{33}})^2.i_3.i_3 \end{pmatrix} \tag{17}$$

the factors $1, i_1, i_2$ and i_3 indicates what is curved and how it is curved.
1 stands for time,i_1 for spacedirection one,i_2 for spacedirection two and ,i_3 for spacedirection three.
Therefore we have similar components like in the Stress Energy Tensor.The imaginaery speeds in the diagonal makes it possible to have negative and positive curvature values.



<u>Hint</u>
The curvaturetensor has as component the Planck Constant which is the constant for quantum physic.Furthermore the curvature is limited if the speed is limited by the speed of light c.

For example if $\phi_{00} = c$ and all other speeds in the tensor $\phi_{xx} = 0$ then

$$K = \frac{c^3}{G\hbar}.\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{l_p^2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{18}$$

$l_p ... Planck\ length$
Now if we remove the indicators $1, i_1, i_2\ and\ i_3$ in tensor K (17) and multiply K by $\frac{c^4}{8\pi G}$ then we get the Energy Stress Tensor T by K

$$T^{\mu\nu} = \frac{c^4}{8\pi G}.K = \frac{c^5}{8\pi G^2\hbar}\begin{pmatrix} (\phi_{00}+i\phi_{\overline{00}})^2 & \phi_{01}\phi_{10} & \phi_{02}\phi_{20} & \phi_{03}\phi_{30} \\ \phi_{01}\phi_{10} & -(\phi_{11}+i\phi_{\overline{11}})^2 & \phi_{12}\phi_{21} & \phi_{13}\phi_{31} \\ \phi_{02}\phi_{20} & \phi_{12}\phi_{21} & -(\phi_{22}+i\phi_{\overline{22}})^2 & \phi_{23}\phi_{32} \\ \phi_{03}\phi_{30} & \phi_{13}\phi_{31} & \phi_{23}\phi_{32} & -(\phi_{33}+i\phi_{\overline{33}})^2 \end{pmatrix} \tag{19}$$

2.2. Calculating the kinetic part of the Lagrangian

$\mathcal{L}_{OQF} = (D^{\mu}\phi)^{\dagger}(D_{\mu}\phi) - V(\phi) \qquad (energydensity)^2$

$D_{\mu}\phi = (\partial_{\mu} + ig\tau_{ij}W_{\mu}^{ij} + ig^{'}I_5 B_{\mu}^0)\phi$

τ_{ij}...*the* $\mathfrak{su}(5)$ *generators see* (14)
W^{ij}, B^0...*the bosons of* $SU(5)$ *and* $SE(1)$
$g, g^{'}$...*the coupling constants*
I_5...5×5 *identity matrices*

The vacuum expectation values VEV are similar to the Higgspotential the minimas of the potential.With basic mathematic like Cardaniac formular and so on we can calculate the value ϕ_{min} see (13) where the potential is a minima.
To get the value for the minima we have to calculate the zeropoints of a cubic equation. The real zeropoints of cubic equations comes from a rotation therefore we have an angle α_{min} in the solution. I just want to give the exact result here.Details see References [W01]

$$\phi_{min} = c.sin(\alpha_{min}).\sqrt[4]{\frac{4\varphi^2}{3}} = c\sqrt{\varphi}\sqrt{cos(\frac{\pi}{6}).sin(\alpha_{min})} = c.\delta \approx c.0,660464 \tag{20}$$

c...*speed of light*
φ...*golden mean* $1,618033...$
$\alpha_{min} = arcsin(\sqrt{cos(\frac{arccos(\sqrt{\frac{3}{4}}^3)+\pi}{3})}) \approx 28,9^o$

$\alpha_{min} \approx \theta_W \quad$ *measured Weinberg angle*

$sin^2(\alpha_{min}) \approx 0,23347$

$\delta = (\varphi \cos(\frac{\pi}{6}).cos(\frac{arccos(\sqrt{\frac{3}{4}}^3)+\pi}{3}))^{\frac{1}{2}} \approx 0,6604642002662$

We have a lot of minimas on the definitionrange the OQF exactly every point of the shape $\mid \phi \mid = \sqrt{\phi^{\dagger}\phi} = \phi_{min}$ is a minima.

We divide the Degrees of Freedome short DOF (see 15) in two parts (we say blue and red).

$$\phi = \begin{pmatrix} i.\phi_0.1 & +\phi_{01}i_1 & +\phi_{02}i_2 & +\phi_{03}i_3 \\ \phi_{10}.1 & +i.\phi_1.i_1 & +\phi_{12}i_2 & +\phi_{13}i_3 \\ \phi_{20}.1 & +\phi_{21}i_1 & +i.\phi_2.i_2 & +\phi_{23}i_3 \\ \phi_{30}.1 & +\phi_{31}i_1 & +\phi_{32}i_2 & +i.\phi_3.i_3 \\ \phi_{\overline{00}}.i.1 & +(\phi_{11}+i\phi_{\overline{11}}).i_1 & +(\phi_{22}+i\phi_{\overline{22}}).i_2 & +(\phi_{44}+i\phi_{\overline{44}}).i_3 \end{pmatrix} + \tag{21}$$

$$\begin{pmatrix} \phi_{2\overline{1}}.1 & +\phi_{\overline{21}}i_4 & +\phi_{\overline{23}}i_5 & +\phi_{\overline{24}}i_6 & +\phi_{\overline{25}}i_7 \\ \phi_{3\overline{1}}.i_1 & +\phi_{\overline{32}}i_4 & +\phi_{\overline{31}}i_5 & +\phi_{\overline{34}}i_6 & +\phi_{\overline{35}}i_7 \\ \phi_{4\overline{1}}.i_2 & +\phi_{\overline{42}}i_4 & +\phi_{\overline{43}}i_5 & +\phi_{\overline{41}}i_6 & +\phi_{\overline{45}}i_7 \\ \phi_{5\overline{1}}.i_3 & +\phi_{\overline{52}}i_4 & +\phi_{\overline{53}}i_5 & +\phi_{\overline{54}}i_6 & +\phi_{\overline{51}}i_7 \\ \phi_{00}.1 & +\phi_{44}i_4 & +\phi_{55}i_5 & +\phi_{66}i_6 & +\phi_{77}i_7 \end{pmatrix} \tag{22}$$

Now we transform like in the Higgs mechanism the violet DOF by (see 15)

$$\phi_{xx}(x) \rightarrow \delta(c + h_{xx}(x)) \tag{23}$$

We see that the second (red) part looks similar to the Higgsfield only instead of 2×2 we have 5×5 fields.
Now we can use the same transformation and results like in the Higgs mechanism.
This means we transform the red part (22) to angle coordinates and the one violet in it by $\phi_{00}(x) \to \delta(c + h_{00}(x))$ (see references [AA01] chapter D).

The local transformation on the red part then is

$$\phi(x) = \delta . e^{i\tau_j \cdot \varphi(x)^j / c} . \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ c + h_{00}(x) \end{pmatrix} \tag{24}$$

with sum up over index j
τ_j*...the* $\mathfrak{su}(5)$ *generators see* (14)
c*...speed of light*
δ*...see* (20)

Now we are doing the inverse gauge transformation then the goldstonebosons disappear.

$$\phi(x) \to \phi(x) . e^{-i\tau_j \cdot \varphi(x)^j / c} = \delta \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ c + h_{00}(x) \end{pmatrix} \tag{25}$$

The total local transformation on the OQF then is (blue and red part together)

$$\phi(x) = \begin{pmatrix} i\delta.(c + h_0).1 & +\phi_{01} i_1 & +\phi_{02} i_2 & +\phi_{03} i_3 \\ \phi_{10}.1 & +i.\delta.(c + h_1).i_1 & +\phi_{12} i_2 & +\phi_{13} i_3 \\ \phi_{20}.1 & +\phi_{21} i_1 & +i.\delta.(c + h_2).i_2 & +\phi_{23} i_3 \\ \phi_{30}.1 & +\phi_{31} i_1 & +\phi_{32} i_2 & +i.\delta.(c + h_3).i_3 \\ i.\phi_{\overline{00}}.1 & +(\phi_{11} + i\phi_{\overline{11}}).i_1 & +(\phi_{22} + i\phi_{\overline{22}}).i_2 & +(\phi_{44} + i\phi_{\overline{44}}).i_3 \end{pmatrix} + \delta . \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ c + h_{00} \end{pmatrix} \tag{26}$$

with h_{xx} *is a* $h(x)_{xx}$ *and* ϕ_{xx} *is a* $\phi(x)_{xx}$

Then for the covariant derivation we get

$$D_\mu \phi = (\partial_\mu + ig\tau_{ij} W_\mu^{ij} + ig' I_5 B_\mu^0)\phi = \tag{27}$$

$$D_\mu \begin{pmatrix} i.\delta.(c + h_0).1 & +\phi_{01} i_1 & +\phi_{02} i_2 & +\phi_{03} i_3 \\ \phi_{10}.1 & +i.\delta.(c + h_1).i_1 & +\phi_{12} i_2 & +\phi_{13} i_3 \\ \phi_{20}.1 & +\phi_{21} i_1 & +i.\delta.(c + h_2).i_2 & +\phi_{23} i_3 \\ \phi_{30}.1 & +\phi_{31} i_1 & +\phi_{32} i_2 & +i.\delta.(c + h_3).i_3 \\ (\delta(c + h_{00}) + i.\phi_{\overline{00}}).1 & +(\phi_{11} + i\phi_{\overline{11}}).i_1 & +(\phi_{22} + i\phi_{\overline{22}}).i_2 & +(\phi_{44} + i\phi_{\overline{44}}).i_3 \end{pmatrix} \tag{28}$$

We are now only interested in the gauge boson masses and therefore set all $h_{xx} = \phi_{xx} = 0$ then

$$D_\mu \phi = (\partial_\mu + ig\tau_{ij} W_\mu^{ij} + ig' I_5 B_\mu^0)\phi = D_\mu \delta.c.i. \begin{pmatrix} 1 \\ i_1 \\ i_2 \\ i_3 \\ -i \end{pmatrix} = D_\mu \langle \phi \rangle \tag{29}$$

$\langle\phi\rangle$...*ground state or vacuum expectation value short VEV*

with (14) we get for

$$\tau_{ij}W_\mu^{ij} = \begin{pmatrix} V^{11} & W^{12}-iW^{21} & W^{13}-iW^{31} & W^{14}-iW^{41} & W^{15}-iW^{51} \\ W^{12}+iW^{21} & V^{22} & W^{23}-iW^{32} & W^{24}-iW^{42} & W^{25}-iW^{52} \\ W^{13}+iW^{31} & W^{23}+iW^{32} & V^{33} & W^{34}-iW^{43} & W^{35}-iW^{53} \\ W^{14}+iW^{41} & W^{24}+iW^{42} & W^{34}+iW^{43} & V^{44} & W^{45}-iW^{54} \\ W^{15}+iW^{51} & W^{25}+iW^{52} & W^{35}+iW^{53} & W^{45}+iW^{54} & V^{55} \end{pmatrix} \quad (30)$$

where the V's are

$V^{11} = W^{11} + \frac{W^{22}}{\sqrt{3}} + \frac{W^{33}}{\sqrt{6}} + \frac{W^{44}}{\sqrt{10}}$
$V^{22} = -W^{11} + \frac{W^{22}}{\sqrt{3}} + \frac{W^{33}}{\sqrt{6}} + \frac{W^{44}}{\sqrt{10}}$
$V^{33} = -2\frac{W^{22}}{\sqrt{3}} + \frac{W^{33}}{\sqrt{6}} + \frac{W^{44}}{\sqrt{10}}$
$V^{44} = -3\frac{W^{33}}{\sqrt{6}} + \frac{W^{44}}{\sqrt{10}}$
$V^{55} = 4\frac{W^{44}}{\sqrt{10}}$

The trace is zero

$V^{11} + V^{22} + V^{33} + V^{44} + V^{55} = 0$

and someone can easy proof that

$(V^{11})^2 + (V^{22})^2 + (V^{33})^2 + (V^{44})^2 + (V^{55})^2 = 2(W^{11})^2 + 2(W^{22})^2 + 2(W^{33})^2 + 2(W^{44})^2$

Therefore the $\tau_{ij}W_\mu^{ij}$ (30) is hermitesch and traceless.
We know that hermitesch matrices are diagonalizable.

we define for $i \neq j$

$$W_-^{ij} := \frac{W^{ij} - iW^{ji}}{\sqrt{2}} \quad and \quad W_+^{ij} := \frac{W^{ij} + iW^{ji}}{\sqrt{2}} \quad and \quad \tilde{V}^{ii} := \frac{V^{ii}}{\sqrt{2}} \quad (31)$$

then

$$\frac{\tau_{ij}W_\mu^{ij}}{\sqrt{2}} = \begin{pmatrix} \tilde{V}^{11} & W_-^{12} & W_-^{13} & W_-^{14} & W_-^{15} \\ W_+^{12} & \tilde{V}^{22} & W_-^{23} & W_-^{24} & W_-^{25} \\ W_+^{13} & W_+^{23} & \tilde{V}^{33} & W_-^{34} & W_-^{35} \\ W_+^{14} & W_+^{24} & W_+^{34} & \tilde{V}^{44} & W_-^{45} \\ W_+^{15} & W_+^{25} & W_+^{35} & W_+^{45} & \tilde{V}^{55} \end{pmatrix} \quad (32)$$

with

$$D_\mu\langle\phi\rangle = (\partial_\mu + ig\tau_{ij}W_\mu^{ij} + ig' I_5 B_\mu^0)\langle\phi\rangle \quad (33)$$

and with (30) and (23) we get

$$D_\mu\langle\phi\rangle = i\sqrt{2}\begin{pmatrix} g\tilde{V}^{11}+g'B^0 & gW_-^{12} & gW_-^{13} & gW_-^{14} & gW_-^{15} \\ gW_+^{12} & g\tilde{V}^{22}+g'B^0 & gW_-^{23} & gW_-^{24} & gW_-^{25} \\ gW_+^{13} & gW_+^{23} & g\tilde{V}^{33}+g'B^0 & gW_-^{34} & gW_-^{35} \\ gW_+^{14} & gW_+^{24} & gW_+^{34} & g\tilde{V}^{44}+g'B^0 & gW_-^{45} \\ gW_+^{15} & gW_+^{25} & gW_+^{35} & gW_+^{45} & g\tilde{V}^{55}+g'B^0 \end{pmatrix} .\delta.c.i \begin{pmatrix} 1 \\ i_1 \\ i_2 \\ i_3 \\ -i \end{pmatrix} \quad (34)$$

then

$$D_\mu\langle\phi\rangle = i\sqrt{2}\delta.c.i.\begin{pmatrix} 1.(g\tilde{V}^{11}+g^{'}B^0)+i_1.gW_-^{12}+i_2.gW_-^{13}+i_3.gW_-^{14}-i.gW_-^{15} \\ 1.gW_+^{12}+i_1.(g\tilde{V}^{22}+g^{'}B^0)+i_2.gW_-^{23}+i_3.gW_-^{24}-i.gW_-^{25} \\ 1.gW_+^{13}+i_1.gW_+^{23}+i_2.(g\tilde{V}^{33}+g^{'}B^0)+i_3.gW_-^{34}-i.gW_-^{35} \\ 1.gW_+^{14}+i_1.gW_+^{24}+i_2.gW_+^{34}+i_3.(g\tilde{V}^{44}+g^{'}B^0)-i.gW_-^{45} \\ 1.gW_+^{15}+i_1.gW_+^{25}+i_2.gW_+^{35}+i_3.gW_+^{45}-i.(g\tilde{V}^{55}+g^{'}B^0) \end{pmatrix} \tag{35}$$

and

$$(D^\mu\langle\phi\rangle)^\dagger = -i\sqrt{2}\delta.c.(-i).\begin{pmatrix} 1.(g\tilde{V}^{11}+g^{'}B^0)+\overline{i_1}.gW_+^{12}+\overline{i_2}.gW_+^{13}+\overline{i_3}.gW_+^{14}-\overline{i}.gW_+^{15} \\ 1.gW_-^{12}+\overline{i_1}.(g\tilde{V}^{22}+g^{'}B^0)+\overline{i_2}.gW_+^{23}+\overline{i_3}.gW_+^{24}-\overline{i}.gW_+^{25} \\ 1.gW_-^{13}+\overline{i_1}.gW_-^{23}+\overline{i_2}.(g\tilde{V}^{33}+g^{'}B^0)+\overline{i_3}.gW_+^{34}-\overline{i}.gW_+^{35} \\ 1.gW_-^{14}+\overline{i_1}.gW_-^{24}+\overline{i_2}.gW_-^{34}+\overline{i_3}.(g\tilde{V}^{44}+g^{'}B^0)-\overline{i}.gW_+^{45} \\ 1.gW_-^{15}+\overline{i_1}.gW_-^{25}+\overline{i_2}.gW_-^{35}+\overline{i_3}.gW_-^{45}-\overline{i}.(g\tilde{V}^{55}+g^{'}B^0) \end{pmatrix}^T \tag{36}$$

then

$(D^\mu\langle\phi\rangle)^\dagger(D_\mu\langle\phi\rangle) =$
$= 2.c^2.\delta^2.[(1.(g\tilde{V}^{11}+g^{'}B^0)-i_1.gW_+^{12}-i_2.gW_+^{13}-i_3.gW_+^{14}+i.gW_+^{15}).$
$(1.(g\tilde{V}^{11}+g^{'}B^0)+i_1.gW_-^{12}+i_2.gW_-^{13}+i_3.gW_-^{14}-i.gW_-^{15})+...] =$

$= 2.c^2.\delta^2.[(g\tilde{V}^{11}+g^{'}B^0)^2+(g\tilde{V}^{22}+g^{'}B^0)^2+(g\tilde{V}^{33}+g^{'}B^0)^2+(g\tilde{V}^{44}+g^{'}B^0)^2+(g\tilde{V}^{55}+g^{'}B^0)^2$
$+g^2 2W_-^{12}.W_+^{12}+g^2 2W_-^{13}.W_+^{13}+...]$

with $W_-^{ij}.W_+^{ij} = \left|W_-^{ij}\right|^2 = \left|W_+^{ij}\right|^2 \quad for\ 1\leqslant i<j\ \leqslant 5$

$$(D^\mu\langle\phi\rangle)^\dagger(D_\mu\langle\phi\rangle) = 2.c^2.\delta^2.[\sum_{1\leqslant i\leqslant 5}(g\tilde{V}^{ii}+g^{'}B^0)^2 \ + \ g^2.\sum_{1\leqslant i<j\leqslant 5}\left|W_-^{ij}\right|^2+\left|W_+^{ij}\right|^2] \tag{37}$$

with

$$\sum_{1\leqslant i\leqslant 5}(g\tilde{V}^{ii}+g^{'}B^0)^2 = \sum_{1\leqslant i\leqslant 5}g^2(\tilde{V}^{ii})^2+\sum_{1\leqslant i\leqslant 5}2g.g^{'}.\tilde{V}^{ii}.B^0+\sum_{1\leqslant i\leqslant 5}g^{'2}(B^0)^2 \tag{38}$$

and $\sum\limits_{1\leqslant i\leqslant 5}V^{ii}=0$ see after (30)

$$\sum_{1\leqslant i\leqslant 5}(g\tilde{V}^{ii}+g^{'}B^0)^2 = \sum_{1\leqslant i\leqslant 5}g^2(\tilde{V}^{ii})^2+\sum_{1\leqslant i\leqslant 5}g^{'2}(B^0)^2 \tag{39}$$

and with $\sum\limits_{1\leqslant i\leqslant 5}(V^{ii})^2 = 2(W^{11})^2+2(W^{22})^2+2(W^{33})^2+2(W^{44})^2$ see after (30)

$$\sum_{1\leqslant i\leqslant 5}(g\tilde{V}^{ii}+g^{'}B^0)^2 = g^2\left|W^{11}\right|^2+g^2\left|W^{22}\right|^2+g^2\left|W^{33}\right|^2+g^2\left|W^{44}\right|^2+\sum_{1\leqslant i\leqslant 5}g^{'2}(B^0)^2 \tag{40}$$

then (37) is in total

$$\underline{(D^{\mu}\langle\phi\rangle)^{\dagger}(D_{\mu}\langle\phi\rangle)} = 2g^2.c^2.\delta^2.$$
$$(\left|W_{-}^{12}\right|^2 + \left|W_{+}^{12}\right|^2 + \left|W_{-}^{13}\right|^2 + \left|W_{+}^{13}\right|^2 +$$
$$\left|W_{-}^{14}\right|^2 + \left|W_{+}^{14}\right|^2 + \left|W_{-}^{15}\right|^2 + \left|W_{+}^{15}\right|^2 +$$
$$\left|W_{-}^{23}\right|^2 + \left|W_{+}^{23}\right|^2 + \left|W_{-}^{24}\right|^2 + \left|W_{+}^{24}\right|^2 +$$
$$\left|W_{-}^{25}\right|^2 + \left|W_{+}^{25}\right|^2 + \left|W_{-}^{34}\right|^2 + \left|W_{+}^{34}\right|^2 +$$
$$\left|W_{-}^{35}\right|^2 + \left|W_{+}^{35}\right|^2 + \left|W_{-}^{45}\right|^2 + \left|W_{+}^{45}\right|^2 +$$
$$\left|W^{11}\right|^2 + \left|W^{22}\right|^2 + \left|W^{33}\right|^2 + \left|W^{44}\right|^2) + 10g^{'2}.c^2.\delta^2.\left|B^0\right|^2$$

$c...speed\ of\ light$
$\delta...see$ (20)

If B^0 is directly the Graviton then it has no mass or massdensity and then $g^{'} = 0$
If $g^{'} <> 0$ then the Graviton must be like in the Elektro Weak Theory a mixing of B^0 and the neutral W^{ii}.

Now what does the factor $2g^2.c^2.\delta^2$ in front of the above equation mean?
We know g is the couplingstrength and $c^2.\delta^2$ is the squared speed value of the miniumum of the potential.Means $\phi_{min}^2 = c^2.\delta^2$
Further we know $g.c.\delta.\left|W^{ij}\right|$ is an energydensity.
If g is dimensionless (gauge theory) then $\left|W^{ij}\right|$ is consequently a pulse density.

2.3. Taking the Evolutionpotential as a characteristic polynomial of a linear map

We can write the Evolutionpotential EP (13) by factorization as characteristical polynomial.
For simplification we set the speed of light c=1 then

$$V(\phi) = -(\frac{\Lambda}{8\pi G})^2.\frac{1}{\varphi^3+1}.(\mid\phi\mid^2 - 0^2).(\mid\phi\mid^2 - 1^2).(\mid\phi\mid^2 - \sqrt{\varphi}^2).(\mid\phi\mid^2 + \varphi^2) \quad (41)$$

$\mid\phi\mid = 0\ and\ \mid\phi\mid = 1\ and\ \mid\phi\mid = \sqrt{\varphi}$ are the zeropoints.

To see the connection to a linear map we write it as determinant

$$V(\phi) = -(\frac{\Lambda}{8\pi G})^2.\frac{1}{\varphi^3+1}.\begin{vmatrix} \mid\phi\mid^2 - 0^2 & 0 & 0 & 0 \\ 0 & \mid\phi\mid^2 - 1^2 & 0 & 0 \\ 0 & 0 & \mid\phi\mid^2 - \sqrt{\varphi}^2 & 0 \\ 0 & 0 & 0 & \mid\phi\mid^2 + \varphi^2 \end{vmatrix} \quad (42)$$

then

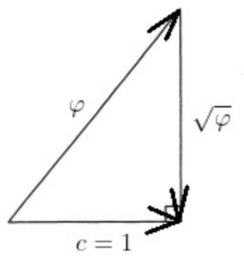
$$V(\phi) = -(\frac{\Lambda}{8\pi G})^2.\frac{1}{\varphi^3+1}.det(\mid\phi\mid^2 .I - M) \quad (43)$$

with I is the unit matrices and M is the matrices for the linear map.

$$M = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \varphi & 0 \\ 0 & 0 & 0 & -\varphi^2 \end{pmatrix} \quad (44)$$

M is traceless and the eigenvalues of M are $0, 1, \varphi, -\varphi^2$ which are squared speeds.
For the eigenvalues $0, 1, \varphi$ we have real speeds $0, 1, \sqrt{\varphi}$ and for the eigenvalue $-\varphi^2$ we have a imaginaery speed $i\varphi$.
So the speed of light c= 1 appears naturally as the root of an eigenvalue.
Furthermore by the trace of M we get

$$1^2 + \sqrt{\varphi}^2 - \varphi^2 = 0 \Leftrightarrow 1^2 + \sqrt{\varphi}^2 = \varphi^2 \tag{45}$$



This picture shows the speed of light as a geometrical result of other speeds
(squareroot of the eigenvalues of M). This triangle in picture (45) is the so called Kepler triangle.
But the physical appearance of the other speeds φ and $\sqrt{\varphi}$ is actually unknown.

3. Conclusion

This paper is just a beginning of understanding Dark Matter and Gravity.
Candidates for Dark Matter are the vectorbosons W of the SU(5) symmetry and
B^0 of the SE(1) symmetry is a candidate for a Graviton.In our picture a Graviton is a particle which is raising the speed of spacetime floating because spacetime curvature is in direct relation to spacetime floating.See (16).Still many questions remain unanswered in this paper.
For example:
Is there a relation between the Evolutionpotential EP (13) and the Higgspotential HP?
If there is a relation between EP and HP then is the angle α_{min} (20) the Weinberg angle?
Is the mass of the SU(5) vectorbosons the Planckmass?

References

[DE01]
PANTELIS A. DAMIANOU AND CHARALAMPOS A. EVRIPIDOU
https://arxiv.org/pdf/1409.3956.pdf

[JM01]
JON MCCAMMOND
https://arxiv.org/pdf/1312.7777.pdf

[KN01]
KIEN NGUYEN
https://www.theorie.physik.uni-muenchen.de/lsfrey/teaching/archiv/sose_09/rng/higgs_mechanism.pdf

[AA01]
AHMED ABOKHALIL
https://arxiv.org/pdf/2306.01019v1.pdf

[RK01]
REINHARD KRONBERGER
https://vixra.org/pdf/2212.0122v5.pdf

[W01]
Geometric interpretation of the roots
https://en.wikipedia.org/wiki/Cubic_equation